Presentation of research achievements

1. First Name and Surname

   Mariusz Białecki

2. Diplomas, degrees - with the name, place and year of obtaining the title of the doctoral dissertation.

   a) M.Sc. in physics, 1996, Institute of Theoretical Physics, University of Warsaw, title "Integrable kinematics of discrete curves", supervisor prof. Antoni Sym;
   b) Ph.D. in physics, 2004, Institute of Theoretical Physics, University of Warsaw, title "Methods of algebraic geometry over finite fields in the construction of integrable cellular automata", supervisor dr. hab. Adam Doliwa (now Professor).

3. Information on employment in scientific institutions

   a) 1996 – 2001 PhD studies, Institute for Theoretical Physics, University of Warsaw;
   b) 2001 - 2002 Institute for Theoretical Physics, University of Warsaw;
   c) 2002 - 2003 Institute for Theoretical Physics, University of Bialystok;
   d) From 2002 Institute of Geophysics, Polish Academy of Sciences, from 2004 as assistant professor;
   e) 2005 – 2007 Graduate School of Mathematical Sciences, The University of Tokyo, scholarship Japan Society for the Promotion of Science (Postdoctoral Fellowship Program for Foreign Researchers);

4. Indication of achievement according to the Act on Academic Degrees and Title and Degrees and Title in Art

   a) the title of scientific / artistic achievement

Stochastic cellular automata and their application to modeling selected aspects of seismicity

b) (author / authors, title / titles of publication, year of publication, name of publisher)

[1] Bialecki, Mariusz, Czechowski, Zbigniew, 2010, On a Simple Stochastic Cellular Automaton with Avalanches: Simulation and Analytical Results in DeRubeis, V, Czechowski, Z; Teisseyre, R, (editors) Synchronization AND TRIGGERING: FROM TO FRACTURE EARTHQUAKE PROCESSES - LABORATORY, AND FIELD ANALYSIS Theories, GeoPlanet-Earth and Planetary Sciences, Springer, Pages: 63-75.
DOI: 10.1007/978-3-642-12300-9_5

[2] Czechowski, Z.; Bialecki, M., 2010, Ito Stochastic Macroscopic Equations as Models of Geophysical Phenomena - Construction of the Models on the Basis of Time Series, in DeRubeis, V, Czechowski, Z; Teisseyre, R (editors) AND TRIGGERING Synchronization: FROM TO EARTHQUAKE FRACTURE PROCESSES - LABORATORY, AND FIELD ANALYSIS Theories, GeoPlanet-Earth and Planetary Sciences, Springer, Pages: 77-96.
DOI: 10.1007/978-3-642-12300-9_6

[3] Czechowski, Zbigniew; Bialecki, Mariusz, 2012, Three-level description of the domino cellular automaton, JOURNAL OF PHYSICS A-MATHEMATICAL AND THEORETICAL Volume: 45 Issue: 15 Article Number: 155101
DOI: 10.1088/1751-8113/45 / 15/155101

[4] Czechowski, Zbigniew; Bialecki, Mariusz, 2012, Ito equations out of domino cellular automaton with efficiency parameters, Geophysica ACTA Volume: 60 Issue: 3 Pages: 846-857.
DOI: 10.2478/s11600-012-0021-0

[5] Bialecki, Mariusz, Czechowski, Zbigniew, 2013, On One-to-One Dependence of Rebound Parameters on Statistics of Clusters: Exponential and Inverse-Power Distributions Out of Random Domino Automaton, JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN, Volume: 82 Issue: 1 Article Number: 014003
DOI: 10.7566/JPSJ.82.014003

[6] Bialecki, Mariusz, 2012, Motzkin numbers out of Random Domino Automaton, PHYSICS LETTERS A Volume: 376 Issue: 45 Pages: 3098-3100
DOI: 10.1016/j.physleta.2012.09.022

c) Discussion of the above-mentioned research work, together with a discussion of their possible use.

A. Introduction

In geophysics, as well as in other natural sciences, observations are of crucial importance. However, direct observations of relevant processes for many geophysical phenomena are beyond our reach. This is the case, among others, of seismology – earthquakes' hypocenters are located too deep to be reached directly in order to make the appropriate measurements. While the phenomena associated with the propagation of seismic waves are relatively well understood - thus we can interpret the seismograms records to reconstruct (a posteriori) histories of faults – but our knowledge of the physical processes leading to successive shifts on a fault during an earthquake initiation is still quite limited. Although the mechanics of destruction is quite a well-developed field of science, however, in the event of seismic processes we are dealing with an extremely complex task. Areas of tectonic faults are substantially diversified, full of cracks and dislocations, for which distributions of size and location are unknown. Thus, stress distributions inside media and the form of a friction law on the fault are unknown too. Also, a role of other processes, such as plasticity, migration of fluids, chemical reactions, etc. is not known. In particular, interactions between them also remain unclear [Corral 2007, Rundle et al. 2003, Rundle et al. 2009]. Information about all of these processes is crucial for the correct description of the process of destruction of the complex geological medium with complex mathematical and physical models. At the same time, it should be stressed, boundary and initial conditions are also unknown. It is also important that, in contrast to many other natural sciences, the ability to conduct experiments for earthquakes is very limited.

On the other hand, earthquakes are very complex phenomena, and it is very difficult to fully reflect their complexity in theoretical models. As well as for other complex geophysical phenomena, their models introduce considerable simplifications in order to control their physical/mathematical structure. However, even substantial simplifications do not guarantee obtaining of transparent formulas, nor exact solutions.

Earthquakes' models largely belong to the domain of complex systems [Newman 2011]. Due to difficulties, mentioned above, stochastic models are widely considered [Rundle et al. 2003]. Introduction of stochastic aspects helps to avoid problems of complex interactions, if they compose in a proper manner. However, there is no way to eliminate another feature of complex systems, which make their description complicated, namely, nonlinearity. In this context, even relatively simple-looking equations can lead to significant difficulties in solving them, or even in analysis of them. For this reason, any links of models with other disciplines are helpful, because they enable to use their various methods.

For those complex phenomena whose physical nature is difficult to understand and explain on the basis on the equations of physics, it appears that cellular automata turn out to be good models [Bhattacharyya and Chakrabarti 2007]. Cellular automata - completely discrete dynamical systems - are defined by specifying the geometry of the system and giving the rule of evolution. Basically, these two components encode essential properties of the modeled phenomena. The rule that defines the automaton can be very simple, but it can lead to very complex evolution of the system and produce a rich structure (patterns), often resulting from nonlinear interactions present in the system.

In the context of applications of cellular automata to modeling complex phenomena, they can be divided into two separated fundamental classes. The aim of the first is to capture the essential features of the phenomenon, while neglecting the majority of "irrelevant" properties. The most important task in this class is an "understanding" the phenomenon. Models in the second class are designed to reconstruct the actual data. Properly tested models of the second type are used for planning and forecasting. Often they are created by the significant completion of models of the first type [Vere-Jones 2009, Newman 2011].

Cellular automata, and more specifically the rules that defines the evolutions, are sometimes treated as a kind of replacement of equations, or even as something opposite to equations - if evolution is given by a simple rule, then equations of evolution may seem unnecessary. There is also another approach, shared by the author of the presented work, taking advantage of both analytical properties of equations and simulations of automata governed by the respective rules. This approach is natural, when the automata are constructed as a counterpart of a particular equation (i.e. Boltzmann equation) [Chopard and Droz 2005].

It should be noted here, that the problem of transformation of continuous models (equations) to the discrete domain is not simple to perform. Substituting the discrete counterparts of derivatives into equations usually results in canceling out important properties. In particular, this is the case of non-trivial complex systems, which exhibit self-similarity property. An example of a paper explaining the subtleties of this topic is the work of [Nishiyama and Tokihiro 2011]. Difficulties associated with finding discrete counterparts of continuous equations, while maintaining their properties are thoroughly examined in the theory of integrable systems (see point 5 A below).

Models presented in the papers described below can be interpreted as a substantial extension of the well-known model of forest fires [Drossel and Schwabl 1992, Malamud et al. 1998]. Similar methods of analysis of equations were used in paper [Paczuski and Bak 1993], although, as is discussed in more detail in [3] and [5], both the context and the purpose of the analysis was completely different.

In the literature there are many works devoted to applications of cellular automata to investigating seismicity. Except of a vast diversity of works related to the self-organizing criticality [Bak et al. 1988], also publications [Vazquez-Prada et al. 2002, Gonzalez et al. 2006, Tejedor et al. 2008, Tejedor et al. 2009, Tejedor et al. 2010] certainly deserve to be mentioned. Those works consider cellular automata, and apply it to the analysis of the various selected aspects of seismicity. They use concepts and methods similar to those applied in papers of this habilitation thesis.

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B. Discussion of the works included in the academic achievement

[1] Bialecki, Mariusz, Czechowski, Zbigniew, 2010, On a Simple Stochastic Cellular Automaton with Avalanches: Simulation and Analytical Results in DeRubeis, V, Czechowski, Z; Teisseyre, R, (editors) Synchronization AND TRIGGERING: FROM TO FRACTURE EARTHQUAKE PROCESSES - LABORATORY, AND FIELD ANALYSIS Theories, GeoPlanet-Earth and Planetary Sciences, Springer, Pages: 63-75. DOI: 10.1007/978-3-642-12300-9_5

In this work we have two objectives: to propose a new model of earthquakes in a form of stochastic cellular automaton and to develop direct analytical approach to the description of the model. Thus, the paper outlines the concepts and methods developed in the subsequent positions of the present cycle.

In order to study fundamental aspects of seismicity there was introduced a simple model that reflects only two properties of earthquakes. The first one is the process of accumulation of energy, assuming that it is supplied to the system at a constant (average) rate. This property, in a simplified way, reflects slow growth of stress caused by the relative movement of tectonic plates. The second feature is the occurrence of an abrupt release of energy - the occurrence of avalanches in the context of the automaton - as it happens, when the stress exceeds in a certain place a given threshold and an earthquake is triggered. This is an example of slowly driven system.

Regardless of the other aspects of the investigated model, it should be emphasized that it appears very interesting opportunity here to look for an answer to the following: which properties of seismicity can be explained by these two above-mentioned assumptions only.

In order to construct the model and incorporate into it those two basic characteristics, the paper [1] establishes geometry and rules of evolution for the automaton with stochastic mechanism for "pumping" energy. The model has to meet two criteria: be simple enough to allow a description by the equation, yet be complex enough to give a wide range of behaviors.

Analytical description of the statistically stationary state of automaton (hereinafter referred to as just stationary) was developed. Balance equations were derived for density, for the total number of clusters and for the function of the number of clusters with a given size. The distribution of cluster sizes leads to distribution of avalanches different from the Gutenberg-Richter law. Balance equations for the moments of clusters' distribution as well as formulas for the average size of clusters and avalanches were derived. Respective formula, which approximates the distribution of clusters were proposed.

For the purpose of analytical considerations an assumption of independence of the clusters was made. This assumption is a kind of extension of the approach used for the study of percolation [Stauffer and Aharony 1992] (in which the cells are independent). The adequacy of this approach was tested both by comparing to the percolation approximation and by comparing to the results of numerical experiments. Agreement of the analytical results with the simulation results is very good, much better than for those obtained within the percolation approximation.

There were also performed various numerical experiments. They showed good agreement of the simulation results with those obtained from the balance equations. Moreover, by means of simulation, it was shown that the analytically derived distribution function of the size of cluster describes the model well, even for densities different from the stationary one.

Finally, there were proposed some possible generalizations of the model related to both the geometry and the rules of evolution.

[2] Czechowski, Z.; Bialecki, M., 2010, Ito Stochastic Macroscopic Equations as Models of Geophysical Phenomena - Construction of the Models on the Basis of Time Series, in DeRubeis, V, Czechowski, Z; Teisseyre, R (editors) AND TRIGGERING Synchronization: FROM TO EARTHQUAKE FRACTURE PROCESSES - LABORATORY, AND FIELD ANALYSIS Theories, GeoPlanet-Earth and Planetary Sciences, Springer, Pages: 77-96.
DOI: 10.1007/978-3-642-12300-9_6

The work is considering a rich and important class of scalar stochastic processes, which can be described by diffusion Markov processes [Oksendal 1998]. Aim of this work is to show, that the Ito equation may be a macroscopic model of the phenomenon, for which the microscopic interactions average properly. Here, cellular automata play the role of fully controllable counterparts of natural phenomena, because the evolution of the automaton - as opposed to natural phenomena - is fully available for observations on all levels.

In order to show that the Ito equation may constitute a good description of the macroscopic behavior of complex systems the following steps were performed. To understand the macroscopic functions present in the Ito equation by means of microscopic rules of cellular automata, there were introduced and analyzed three different kinds of cellular automata. Using the obtained results, certain aspects of the microscopic behavior of the geophysical process - variation of optical thickness of aerosols in the atmosphere - were explained. There were derived respective formulas for the Ito equation describing the macroscopic behavior of the proposed in [1] stochastic automaton domino expressed by the distribution functions of clusters. Derivation was due to the good properties of the previously obtained equations describing the automaton. As a first approximation for the distribution of clusters as a function of the density it was taken in a stationary state form. It provides a quite good agreement with the Ito equation, numerically reconstructed on the basis of time series generated by the automaton using the histogram method [Siegert et al. 1998]. It should be noted that the Ito equation describes the behavior of selected macroscopic variable (i.e. density) over time. This means that in addition to stochastically stationary balance equations we have also the equation of evolution.

Moreover, there were presented some attempts to control stochastic phenomena described by the Ito equation in order to reduce the likelihood of extreme events.

[3] Czechowski, Zbigniew; Bialecki, Mariusz, 2012, Three-level description of the domino cellular automaton, JOURNAL OF PHYSICS A-MATHEMATICAL AND THEORETICAL Volume: 45 Issue: 15 Article Number: 155101
DOI: 10.1088/1751-8113/45/15/155101

Development of ideas about the modeling of complex phenomena using Ito equation resulted in a three-level description of a complex system, modeled by the domino cellular automaton. The inspiration for such a description is the kinetic theory of gases, which distinguishes a microscopic level (the collisions of particles with a description by a huge system of Newton equations), a mezoscopic level (particle velocity distribution function, described by integral-differential Boltzmann equation) and a macroscopic level (description by the moments of the velocity distribution function: the gas density, macroscopic velocity, pressure and temperature). By determining subsequent moments for Boltzmann equation there are obtained macroscopic equations describing the gas: the continuity equation, Navier-Stokes equation and the equation of heat transfer. In this way, higher levels have a clear interpretation in terms of the lower levels [Cercignani 1975].

This paper presents a three-level description of the stochastic automaton domino as follows. Microscopic description is given by fixed geometry and the set of rules of evolution of the automaton. Mezoscopic description is given by the equations for the size distribution function of clusters. Macroscopic description consists of equations for the moments of this distribution and the corresponding Ito equations describing the time evolution of density of cellular automaton.

The introduction of such a description required implementation of a series of smaller tasks. In particular, a method of determining the distribution of clusters as a function of lattice density, i.e. for states with various densities deviated from the steady state density. The obtained relations were used to determine the moments as a function of density. It was observed a very good agreement with the results of the simulation.

Completion of the macroscopic description by the equation for evolution of the density was made by derivation of relevant Ito equation using the obtained cluster distributions as a function of density. This is much more precise method than for stationary formulas as in [2]. There is a good agreement of this result with the Ito equation numerically reconstructed on the basis of time series generated by the automaton. Moreover, the statistical characteristics of time series generated by the automaton itself, and by derived Ito equation showed a very good agreement, which means that the Ito equation is a good description of the macroscopic evolution of the density oscillations of cellular automaton.

The meaning of this work relies on determining the way and developing respective methods for the transition from microscopic to macroscopic description for stochastic cellular automata which are simple models of earthquakes. The importance of this problem in the case of the description of the gas is widely known.

[4] Czechowski, Zbigniew; Bialecki, Mariusz, 2012, Ito equations out of domino cellular automaton with efficiency parameters, Geophysica ACTA Volume: 60 Issue: 3 Pages: 846-857.
DOI: 10.2478/s11600-012-0021-0

This work is a continuation and an expansion of the program outlined in [3] for a bigger class of cellular automata. The main result is a derivation of the Ito equations for the extended domino cellular automaton with efficiency parameters.

There was introduced a cellular automaton with efficiency parameters - quantities associated with the probability of energy dissipation for both the occupied and empty cells. In previous work probabilities were equal to zero. Using various values of these parameters one can obtain various values for steady state densities.

For such a generalized system, appropriate numerical simulations were performed and respective balance equations, formulas on the mean size of the clusters and avalanches, and the equation for the moments were derived. There were numerically examined the following quantities: the density of the steady state, the total number of clusters, the average size of clusters and avalanches and the analogue of temperature, all of them as a functions of the ratio of the efficiency parameters.

There was also modified the analytical method to derive the Ito equations for automata, in order to take into account the influence of efficiency parameters. To achieve this goal, there was set the appropriate formula describing the deviation from the steady-state and there was made necessary rescaling of the efficiency parameters.

The obtained analytical results were used to compare the results of histogram reconstruction method for Ito equation for three different cases with different size parameters of efficiency (and therefore density). A good consistency of both methods is an argument for the adequacy of Ito equation to describe this extended class of processes.

[5] Bialecki, Mariusz, Czechowski, Zbigniew, 2013, On One-to-One Dependence of Rebound Parameters on Statistics of Clusters: Exponential and Inverse-Power Distributions Out of Random Domino Automaton, JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN, Volume: 82 Issue: 1 Article Number: 014003
DOI: 10.7566/JPSJ.82.014003

This subsequent work of the present series focuses on the properties of the cellular automaton. The proposed substantial extension preserves the "good" mathematical structures, used in a significant way in the previous works. A range of properties of time series generated by the automata has also been relatively enlarged.

The generalization is obtained by replacement of efficiency parameters discussed above (which are constant and express respective probabilities) by general functions dependent on the size of clusters. In this case, the probability of energy dissipation depends in a desired manner on local properties of the location where the energy is to be stored.

The main objective of the paper is to show the direct existence of a one-to-one correspondence between the efficiency parameters and the function of distribution of clusters and, consequently, distribution of avalanches generated by the automaton. Non-trivial task related to this relationship is to show how to infer about properties of the microscopic interactions (i.e. about functional form of the efficiency parameters) from a given distribution of clusters.

This result concerns the fundamental problem of reconstruction of actual physical processes on a fault based on the observed distribution of earthquakes. In addition to the development of generalized domino cellular automaton with efficiency parameters dependent on the size of clusters and investigation of numerical simulations for a variety of their forms, the work contains many new results.

There were derived general balance equations for valid for all efficiency parameters in the form of function described above. The resulting equations enabled the derivation of a hierarchy of equations for the moments (of any level). Analysis of the form of this hierarchy of equations allowed selecting some interesting cases, including the exponential and inverse-power ones. Thus, this work put in a broader context the previously obtained case with constant efficiency parameters (leading to the exponential distribution of sizes of clusters).

It was interesting to study the behavior of the automaton with an inverse power form of efficiency parameters. This case leads to long-tail (asymptotically inverse-power) distributions for cluster sizes.

Despite of its simplicity, generalized domino cellular automaton is characterized by high universality. The paper presents examples of the automaton as a model for the interpretation of various geophysical phenomena (earthquakes, forest fires) and also its application to testing the reconstruction procedure of Ito equation. It is also mentioned an important relationship with known combinatorial object - further details are presented in paper [6].

The Appendix presents the derivation procedure of the balance equations for so-called empty clusters, and their moment of order zero. They are indispensible for closing the system of equations for the balance of clusters.

It should be emphasized that the presented generalized version of cellular automaton reconstruct Gutenberg-Richter law, as well as (any) deviations from this law.

[6] Bialecki, Mariusz, 2012, Motzkin numbers out of Random Domino Automaton, PHYSICS LETTERS A Volume: 376 Issue: 45 Pages: 3098-3100 DOI: 10.1016/j.physleta.2012.09.022

In this work, some theoretical topics discovered during investigating of properties of the general form of stochastic domino automaton are studied.

There was examined the case of inverse-power form of efficiency parameters, which allows to write the system of equations describing the steady state of the automaton in the form of a recursion, commonly known in combinatorics. The recursion, for appropriate initial conditions, leads to the so-called Motzkin numbers [Sloane and Plouffe 1995]. Motzkin numbers belong to the class that contains also the Catalan numbers, which are the most common object in combinatorial mathematics (there are dozens of different interpretations of the Catalan numbers). Catalan numbers are given by the recurrence of the first order, while the Motzkin numbers are given by recursion of the order two [Aigner 2007, Flajolet and Sedgewick 2008]. The work [6] has established for the first time the relationship between Motzkin numbers and a subject of stochastic cellular automata.

Therefore, the detected link to combinatorial objects is helpful – it allows the use of well-developed combinatorial methods for the analysis of stochastic domino automaton. In particular, there was used the method of generating functions [Aigner 2007, Flajolet and Sedgewick 2008] to determine the explicit form of the general solution of the recursion (for any initial condition), and thus to find formulas for the distribution of cluster for the automaton in this case. Finding explicit form of solutions for nonlinear problems is usually recognized as a remarkable achievement.

Moreover, using the knowledge of the asymptotic behavior of Motzkin numbers, there was shown analytically, that under certain conditions, the distribution of clusters is asymptotically inverse-power with an exponent equal to 3/2.

C. Use of the results

The above results have been used to further development of both the description of stochastic cellular automata and the analysis of selected aspects of seismicity. In particular, the extension of the results of the work [5] is in preprint [Bialecki 2012] (published in the archive arXiv), whose rich material has been prepared for publication. Praca [Bialecki 2013] presents a detailed reconstruction procedure of the rules governing the microscopic dynamics of the automaton (i.e. the efficiency parameters) on the basis of (any) specified distribution of avalanches for finite cellular automaton. Based on the described results there is one another work [Bialecki 2012b], available now in the form of a preprint (published in the archive arXiv). This last article presents an explanation of the universal of curve for earthquakes recurrence time. It is a direct confirmation, that the domino cellular automaton is useful to clarify important issues of seismology. Below I will discuss in turn (in a logical order) all the obtained results on the application of the stochastic cellular automaton.

Finite domino automaton [Bialecki 2012a] was created to examine the assumptions of the analytical description of the domino automaton. In particular, to determine the size of the correlations and to perform comparison of the accurate values obtained within the framework of Markov chains with those obtained from the balance equations.

For this purpose, it was necessary to develop an analytical description of the finite version of stochastic domino automaton. Previous considerations of domino automaton were performed in the so-called thermodynamic limit, i.e. under assumption that the system is suitable enough for the boundary conditions do not affect the properties of the automaton. In the following, the analytical description incorporates the effect of finite size.

Similarly as in [5], under assumption of independence of clusters, there were derived respective equations describing the steady-state cellular automaton, namely: the equation of balance of density, the total amount of clusters and the number of clusters of a certain size. It was shown that some of the equations are exact, i.e. they include all occurring correlations. There was also shown, that certain factors (of combinatorial origin) can not be exactly expressed by the variables used in the equations for the size of the automaton greater than four, and there were proposed the corresponding approximate formulas. Moreover, there was considered the thermodynamic limit of the obtained equations - it is identical to the equations work [5].

The paper presents a description of the automaton as a Markov chain: adequate space of states has been described (irreducible, aperiodic and recurrent). The steady state was precisely defined in terms of the transition matrix. It should be noted, that the terminology of Markov chains enable to directly conclude existence of a steady state - which is not trivial to establish, without this concepts.

There were derived various relations specific to a number of selected cases. Appropriate parameters in the finite cases can be chosen from significantly greater range comparing to the infinite version, because all sequences are finite, and there is no need to impose proper conditions for their convergence.

This enables the consideration of new classes of efficiency parameters, leading to a quasi-periodic evolution of the system. In this way, within the uniform description of domino cellular automaton, one more important new class of automata was constructed.

There were considered expected values of return times between states with empty lattice and also between states with fully occupied lattice. There was shown how to derive the appropriate formulas and some exemplary results were presented in the Appendix. There were derived the formula for the average time between any avalanches, which connect the macroscopic quantities (average size of avalanches) and the microscopic quantities (the probability of rebound of incoming particles representing energy). There are also presented two other formulas for the average time between any avalanches, as well as the method of determining the average time between avalanches of a given size.

The work contain detailed examination of automata of size 3 (with 4 states, the smallest non-trivial system), 5 (8 states, the first for which there are no exact coefficients in the equations), 7 (20 states, for which the actual density distributions were tested) and 10 (108 states, the smallest size, where there is a state containing the same clusters, but in significantly different order - a word "significantly" exclude a change of order obtaining by mirror reflections).

Distributions of clusters were tested for various densities of the system, also for those close to zero and close to unity. The approximate coefficients of equations were compared with the exact values (derived from Markov chains) and their accuracy was evaluated. The analysis leads to the following conclusion: investigated various cases show similar qualitative behavior for sizes 5, 7 and 10. It supports the natural assumption, that the results for small sizes can be transferred (mutatis mutandis) to larger systems. Therefore studies of relatively small systems help to understand behavior of such automata in general.

Based on the above results, paper [Białecki 2013] presents an analytical procedure for the determination of the efficiency parameters characterizing the internal dynamics from a given distribution of avalanches.

There is presented a system of equations (with no a priori unknown efficiency parameters), which allows to determine the distribution of clusters on the basis of the distribution of avalanches. Knowing the distribution of those two kinds of parameters, efficiency parameters can be easily determined.

The proposed procedure was tested in the following way. The automaton of size 5 was prepared with four different distributions of avalanches: exponential, inverse-power with exponents equal to -1 and -2, and a M-shaped distribution of, as an example of fancy distribution. On this basis, corresponding efficiency parameters were determined – in other words, the dynamics leading to these distributions were captured. Further, by using Markov chain techniques, from these efficiency parameters there were determined avalanche distributions in an accurate manner. Then those two distributions of avalanches were compared to each other. It was observed good agreement in all cases, even though the considered examples, due to the small size of system, were inferred by relatively big correlations. This result is in a favor of the proposed method.

Another important conclusion from the results presented above is the observation that the domino automaton allows to explain the shape of the universal curve of recurrence time of earthquakes [Bialecki 2012b].

This curve, introduced in [Corral 2004], is a universal characteristic of seismic activity on the Earth. In order to harmonize the various observations Corral performed the following procedure. Seismicity was considered as the result of dynamic processes that aggregate various properties, and which in result are largely independent of the physical properties of individual earthquakes (see also [Corral 2007]). Following an approach presented in paper [Bak et al. 2002], Corral does not divide earthquakes into different types (foreshock, mainshock, aftershock) nor the Earth's crust is divided into areas with different tectonic properties. Then, for a selected region of the Earth (chosen by taking a range of geographic coordinates), for a selected period of time and for accordingly selected the minimum magnitude threshold, earthquake are characterized by time of the occurrence only (and hence the order). Thus earthquakes are described by the point process (in time). Then, the difference of occurrence time between two consecutive earthquakes (sometimes called a return time) is investigated. After appropriate rescaling by the average seismicity of the region, there is obtained a universal curve for earthquake recurrence time distribution, independent of the choice of the region. This curve is approximated by a generalized gamma distribution, but for small values of recurrence time there are observed deviations from the gamma fit [Corral 2004, Marekowa 2012].

Based on the combinatorial properties of a finite domino automaton, work [Bialecki 2012b] shows both the mechanism leading to the correct shape of the curve and to the formation of deviations for small return times. Notice, many accepted models of earthquakes can not reproduce return time distribution properly [Weatherley 2006].

Thus, on the basis of studies of a simple cellular automaton model of earthquakes, the conclusion has been made, that the shape of the universal curve of earthquake recurrence time distribution is of a combinatorial nature, and it results from the two main characteristics of earthquakes: the slow accumulation of energy and its rapid release (according to the rules implemented in the domino automaton).

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C. Summary

The presented works investigate stochastic cellular automata and their applications to selected aspects of seismicity. Their interdisciplinary features reflect the broad experience of the author, which took place during his professional development. The importance of this work relies on the innovative combination of different disciplines. Recently, this approach has gained some recognition. Together with prof. Zbigniew Czechowski as a leader, we succeeded in obtaining a research project for developing the ideas discussed above. The obtained NCN support - project 2012/05/B/ST10/00598 - guarantees that the results will be further developed.

In a somewhat broader sense, the discussed subject concerns the modeling in general, the topic that is interesting for many geophysicists and other scientists. It was clearly visible, when organizing (jointly with prof. Zbigniew Czechowski) Seminar "Models in Geophysics", which took place from April 2010 to June 2011 at the Institute of Geophysics. As a result of this seminar, there is prepared a topical issue "Advances in Geophysical Processes - Models and Methods," of the journal Acta Geophysica (on the Web of Science list, 20 pts.), in which I am honored to serve as guest editor. Contents of this issue, where majority of papers were submitted by the authors from abroad, suggests that interest in this subject is significant in the world.

5. Presentation of other research achievements.

My other scientific achievements are grouped around two themes. The first one, related to the subject of my doctorate, is a kind of basic research in mathematical physics, which concerns so called integrable cellular automata. The second one, undertaken after employment at the Institute of Geophysics, focuses on the mechanics of asymmetric media.

A. Integrable cellular automata - methods of algebraic geometry, dynamical systems over finite bodies.

A.1. List of publications

[7] Doliwa, A; Bialecki, M; Klimczewski, P, 2003, The Hirota equation over finite-fields: algebro-geometric approach and-multisoliton solutions, JOURNAL OF PHYSICS A-MATHEMATICAL AND GENERAL Volume: 36 Issue: 17 Pages: 4827-4839 DOI: 10.1088/0305-4470/36/17/309

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[12] Bialecki, Mariusz, Nimmo, Jonathan JC, 2007, On pattern structures of the N-soliton solution of the discrete KP equation over a finite field JOURNAL OF PHYSICS A-MATHEMATICAL AND THEORETICAL Volume: 40 Issue: 5 Pages: 949 - 959 DOI: 10.1088/1751-8113/40/5/006

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A.2. Description

In the theory of classical integrable systems, some non-linear differential, differential-difference and difference equations with certain specific properties are considered. Strongly simplifying the issue, despite the fact that there are no general methods for solving nonlinear equations, there is a class of equations (called integrable equations) for which there exist effective methods for generating specific solutions. These solutions include the so-called solitons solutions. Figuratively speaking, integrable equations represent the properties of non-linear equations, which are opposed to the non-linearity associated with deterministic chaos.

The issue of construction of integrable cellular automata was undertaken since the beginning of the nineties of last century. There are two significantly different systematic methods for the construction of such automata. The first, called the ultradiscretization procedure (or limiting procedure), was initiated by the work of [Tokihiro et al. 1996]. Second method, concerning the construction of automata over finite fields, was presented in a series of works [7-11] mentioned above. Works [12-13], have to integrate these two approaches. A more complete review of this subject is contained in my doctoral dissertation [Bialecki 2003].

The basic idea contained in the cycle is as follows. It was known that there exists algebro-geometric method of construction of solutions of continuous and discrete equations [Belokolos et al.]. It is also known that the structure of projective geometry (in particular algebraic geometry) is in general independent of the base field (on which a construction is performed). Thus, by modifying the algebro-geometric methods in an adequate way for finite fields [Lang 1970], we get the method of construction of integrable cellular automata.

Modification of algebro-geometric method for finite fields for the Hirota equation (discrete analog of the generalized Toda equation) is presented in paper [7]. In addition to providing general construction, it contains construction of so called multisoliton solutions, and discusses the class of so-called breather-type solutions, for which there are no equivalents in the field of complex numbers.

In paper [8] there was presented the construction for the discrete Kadomtsev-Petviashvili equation (dKP) over finite fields and the corresponding reduction to the Korteveg-de Vries equation (dKdV) over finite fields. Ther were presented the general formulas for multisoliton solutions (for any non-singular curve). The same method adapted for one-dimensional Toda equation is included in paper [11].

In paper [9], there was emphasized the role of the Jacobian of the curve [Cornel and Silverman 1986] underlying the algebro-geometric construction and there was presented respective construction of a more general solution of the dKP equation for a hyperelliptic curve over finite fields [Griffiths and Harris 1978]. Work [10] shows similar solution for the dKdV equation with a corresponding solution of the dKP equation based on hyperelliptic curve. Similar mathematical techniques are also used in advanced cryptography [Koblitz 1998] and in the theory of codes [Stichtenoth 1993].

Work [12] analyzes the existence and properties of coherent structures (patterns) of multisoliton solutions of the equation DKP over finite fields. Work [13] presents the elements of the Sato theory [Ohta et al. 1988] in the general form valid for any field.

Those publications deals with complicated problems of mathematical physics, and use advanced mathematical techniques. Despite the subject is quite sophisticated, obtained results have been appreciated by experts. In particular, the work [9] appeared in a very prestigious journal Communications in Mathematical Physics. After contacting professor Tetsuji Tokihiro (one of the main authors of the above-mentioned ultradiscretization procedure), I obtained scholarship of Japan Society for the Promotion of Science (Postdoctoral Fellowship for Foreign Researchers) for two-year collaboration with the group in the famous Graduate School of Mathematical Sciences, The University of Tokyo. Also I gave one invited lecture on my work at a conference in the Research Institute for Mathematical Sciences (RIMS), Kyoto University.

Now, several years after publication of described works, the interest in the subject is growing. The articles [7, 8] are cited as pioneering work [Kanki et al. 2012].

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Tokihiro, T., Takahashi, D., Matsukidaira, J., Satsuma, J.: From soliton equations to integrable cellular automata through a limiting procedure. Phys. Lett 76 (1996) 3247-3250

B. Asymmetric continuum theory – rotational deformations, discrete theory of defects

B.1. List of publications

[14] R. Teisseyre, M. Bialecki and M. Górski, 2005, degenerated mechanics in a homogeneous continuum: Potentials for spin and twist, Acta Geophysica POLONICA 53, No. 3 (2005) 219-230.

[15] R. Teisseyre, M. Bialecki and M. Górski, 2006, degenerated Asymmetric Continuum Theory, Chapter 5 in R. Teisseyre, M. Takeo and E. Majewski (Eds.) 'Earthquake Source Asymmetry, Structural Media and Rotation Effects 'Springer, pp. 43-55

[16] M. Bialecki, 2006, Towards a theory of discrete defects, Chapter 7 in R. Teisseyre, M. Takeo and E. Majewski (Eds.) 'Earthquake Source Asymmetry, Structural Media and Rotation Effects ", Springer, pp.. 67-76.

B.2. Description

The works [14] and [15] contain a discussion of degenerate homogeneous continuum mechanics, in which the movements are negligible, and there are only rotational movements (spin and twist). It is the opposite case to the classical description of elasticity. The proposed system of potentials helps to understand the wave movements in such medium and geometrical properties of the degenerate mechanics and its Riemannian geometry.

Work [16] proposes general ideas for the description of defects in the crystal-like media via discrete equations, instead of commonly used continuous description. It also gives arguments in favor of such a description. As an example of the proposed approach, the paper presents a discrete version of Weingarten Theorem ("jump" of displacement field on the surface of the intersection is in the form of sum of a constant vector plus fixed rotation).

Those works investigate some theoretical concepts that originated from the theory of defects. While the topic presented in paper [16] did not have so far a further development, it is just the opposite in the case of works on asymmetric continuum mechanics. Those results are used in the rapidly growing - both in terms of theory and measurements - rotational seismology. Visible effect of the growing interest in this subject is the number of citations of the work [14] - now the work is cited 8 times by the articles from the list of Web of Science, even though the work itself is not on this list.

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